High Temperatures-High Pressures, Vol. 37, pp. 137–151
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A review of vibrating objects for the measurement of density and viscosity in oilfields including devices fabricated by the method of MEMS

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Received: November 15, 2007. Accepted: December 22, 2007. In Final Form: January 14, 2008.

The thermophysical properties of fluids are required for the design of many industrial processes. In the case of design there is a choice to be made between prediction of the properties of the fluids from physically-based or empirical models and their direct measurement. For fluids and circumstances that is extreme it can be cost-effective to measure the properties directly with accuracy commensurate with the purpose. Process monitoring and thus control can be effective by continuous observation of a thermophysical property of a process stream. In either of these applications the instruments used for these *in situ* measurements must be able to withstand the conditions of measurement, and be robust and stable. Experience has demonstrated that these features can be provided for mechanical properties by the application of vibrating objects with a variety of geometries. In such instruments the complex resonance of a vibrating object immersed in a fluid is measured and then related to the required thermophysical properties using working equations founded on the principles of physics. In this paper we provide three examples of instruments that operate under these premises and yield data with an accuracy that are "fit-for-purpose". These are: (1) a vibrating wire viscometer and measurements of the viscosity of diisodecyl phthalate (a fluid which could be used as a viscometer calibrant at viscosities of the order of 100 mPa·s); (2) the development of MEMS devices to measure both the density and viscosity of fluids with

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uncertainties of ±1% and ±10% respectively; and (3) a MEMS device to measure the non-Newtonian properties of fluids.

Keywords: density, MEMS, microelectromechanical system, diisodecyl phthalate, viscosity, vibrating plate, vibrating objects, vibrating wire.

1 INTRODUCTION

Thermophysicists have devoted considerable effort to both the development of the theory of molecular interactions, and experimental techniques to provide precise values of desired thermophysical properties. Careful measurements on a few selected systems have been used to verify the theoretical models [1]. In the case of the transport properties of fluids this procedure took over two decades to come to fruition. In reference [1] it was argued that, in many 21st century scenarios, there is simply insufficient time to permit this procedure to be followed in every case. For example, in the oil industry two decades is typically similar to the time that it takes to explore, produce and exploit all accessible hydrocarbon from a reservoir. It was also suggested in reference [1] that the fluids whose properties are now required are often quite different from those to which the field of thermophysics has previously been accustomed. Frequently, for example, the chemical composition of the fluid may be unknown so that standard predictive methods have no utility. This situation is encountered in the oil industry, and yet the physical properties of fluids are still required for design calculations and the evaluation of the economic value of a reservoir. Whether motivated by demands of physics or engineering, fluid properties will also be required for fluids in aggressive environments of high temperature and pressure where the fluid itself may also be chemically corrosive. In such difficult circumstances, while the accuracy of any property measurement will need to be known, it may be of secondary importance to having a value of known uncertainty which has been selected to be consistent with the intended use of the results. In the case of process monitoring, and in some cases also control, an instrument is required to monitor a physiochemical property. Instruments used for these *in situ* measurements must be robust and stable. These design and other criteria are, for mechanical properties, satisfied by vibrating objects with a variety of geometries, simply because frequency is a quantity that is easily measured with high precision by electronic components. The complex resonance of a vibrating object is determined and linked to the desired thermophysical properties with working equations founded on exact physical principles; in general the response is proportional to either the quotient or product of density and viscosity. The design of the instrument or the conditions may render high precision in every measurement impossible so that the accuracy secured is limited to that adequate for the need. This leads to the concept of an accuracy of a measurement that is 'fit for purpose'.

In the remainder of this paper, we provide three examples of this concept. The first is a vibrating wire viscometer developed to measure the viscosity of a

material that has been proposed as a reference material for viscosity of the order of 100 mPa·s and measurements with an expanded uncertainty of about ±1%. The second case is the design and application of MicroElectroMechanical System (MEMS) for operation in the petroleum industry and, in particular, to measure both the density and viscosity of hydrocarbon reservoir fluids in the bore-hole. In this case, measurements of density and viscosity with uncertainties of about ±1% and ±10%, respectively, are considered adequate to guide calculations about the value of a reservoir and its exploitation strategy with sufficient rigor. Our third and final example is a MEMS device designed for the measurement of the non-Newtonian properties of fluids.

2 VIBRATING WIRE VISCOMETER FOR DIISODECYL PHTHALATE (DIDP)

Our first example is provided by the development of a viscosity standard for industrial purposes for fluids of moderate viscosity from (10 to 100) mPa·s (equivalent to that of so called conventional hydrocarbon resources). The existence of a viscosity standard that lies within this range at a temperature near ambient removes the requirement to use a chain of reference fluids and a series of Master capillary viscometers in a step-up procedure starting with the viscosity of the international primary standard [2] of water at $T = 293.15$ K and $p = 0.101325$ MPa of (1.0016 ± 0.0017) mPa·s [3]. Every sequential step in this procedure, where the viscosity departs further from that of water, propagates its uncertainty and adds to that of every previous step. In addition, the process is extremely demanding and time consuming in an industrial context [2].

To overcome this difficulty and provide a suitable calibrant fluid the International Association of Transport Properties instigated a project on "Investigation of a New High-Viscosity Standard." and has proposed diisodecyl phthalate (DIDP) as a suitable candidate in the viscosity range (100 to 200) mPa·s because it is a liquid over a wide range of temperatures, is readily available, has a low volatility and is non-toxic [4].

Within this project, the viscosity of DIDP has been measured with a variety of methods including vibrating-wire viscometers. In general, this instrument simply consists of a metallic wire, usually made from tungsten or stainless steel, which is tensioned, either by a mass, suspended from its lower end, or by being clamped between two rigid supports. A current is then passed through the tensioned wire which, in a permanent magnetic field perpendicular to its length, forces it to move. A continuous alternating current can be used (forced vibrations) or a direct current may be applied to displace the wire from its rest position, and the decay of amplitude measured after release (transient). Measurements of the current induced in the wire by its motion in the magnetic field provide the resonance in the forced oscillation mode or the logarithmic decrement and frequency in the transient decay mode. From either measurement method, it is possible to determine the viscosity and when

a mass is suspended from one end it is also possible to obtain density of the fluid using an appropriate model that has been reported elsewhere [5,6].

For measurements with DIDP a wire clamped at both ends and optimised to measure viscosity has been developed [7]. This enables measurements of the viscosity of DIDP with an expanded uncertainty of about $\pm 1\%$, which is about 5 times greater than the uncertainty that can be achieved with a vibrating wire viscometer for fluids with viscosity of the order of 1 mPa·s [8]. The wire radius was determined from measurements in water at a temperature of 293 K, thus the calibration is directly traceable to the international standard for viscosity of water [9,10]. The instrument, illustrated in Figure 1, is really a vibrating-rod viscometer rather than a vibrating- wire viscometer because the diameter of the wire is about 200 μm (rather than the 50 μm used in early versions [8]). Its design is necessarily more robust in order to be able to set such a rod in transverse oscillation in a manner that the reaction of its support is negligible. The physics that describe the motion of the wire

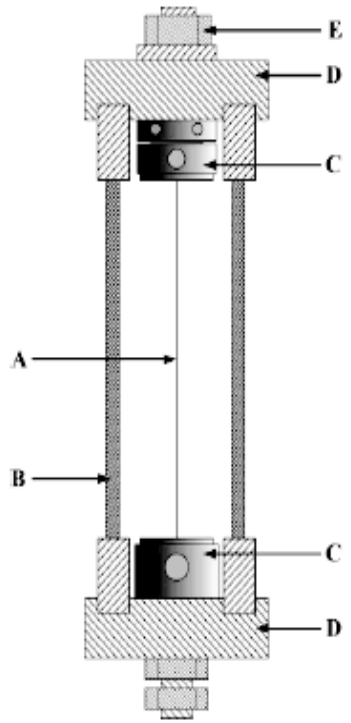

FIGURE 1
Schematic drawing of the vibrating-wire viscometer; A, Tungsten vibrating wire of length about 60 mm; B, Tungsten rod spacers; C, Inconel clampers for the vibrating wire; D, Inconel plates; E, Fine threaded nut, for tensioning without torsion of the wire. Reproduced with permission of the American Chemical Society [7].

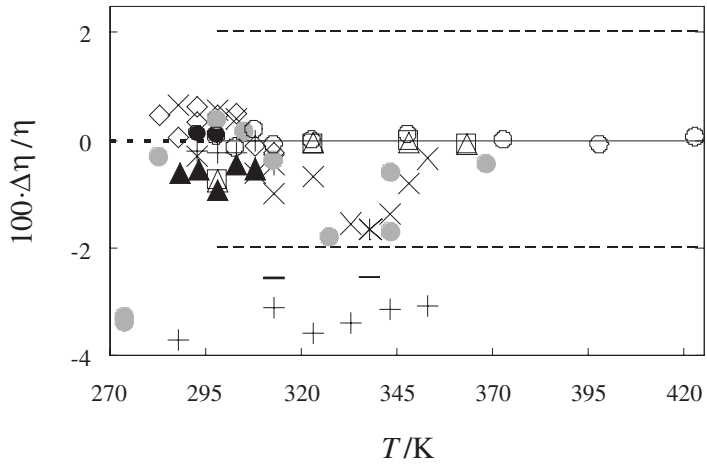

FIGURE 2
Relative differences $\Delta\eta/\eta = \{\eta(\text{expt}) - \eta(\text{calc})\}/\eta(\text{calc})$ of the experimentally determined viscosity η(expt) at $p = 0.1$ MPa for different impurities and water mass fraction w, from the value obtained from equation (4) of reference [11] η(calc) as a function of temperature T. Δ, Al Motari *et al.* [11]; ◯, Al Motari *et al.* [11]; □, Al Motari *et al.* [11]; ▲, Caetano *et al.* [4]; ◇, Caetano *et al.* [9]; •, Caetano *et al.* [10]; +, Harris and Bair [12]; ×, Harris and Bair [12]; ∗, Harris and Bair [12]; −, Harris and Bair [12]; •, Peleties and Trusler [13]. The dashed lines at ±2% are the expanded uncertainties in the measurements of Reference [11] while that at 0 indicates an extrapolation of equation (4) of reference [11] to temperatures below the measurements of Reference [11] to which the parameters were adjusted. Reproduced with permission of the American Chemical Society [11].

remains the same but the precision of the measurements of the motion is less than can be accomplished with a wire. The accuracy of the final property measurement is therefore fit for its purpose rather than the highest that could be obtained. As part of an international collaboration, other workers [11–13] have measured the viscosity of DIDP; reference [11] for example used a vibrating wire clamped at both ends. The deviations of the results of these measurements at a pressure of 0.1 MPa from a Vogel equation [14] are shown in Figure 2. The parameters of the Vogel equation have been adjusted to best represent the results obtained from another vibrating wire viscometer [8] that have a greater expanded uncertainty of ±2%. The deviations from the same correlation of the measurements with a capillary viscometer at $p = 0.1$ MPa reported by Peleties and Trusler [13] are also shown in Figure 2.

Figure 3 shows the results reported in the literature [11,12] at pressures greater than 0.1 MPa as deviations from a smoothing equation described in reference [11]. The vibrating wire measurements of reference [11] were reported at temperatures from (298 to 423) K and pressures between (0.1 and 70) MPa while those values reported by Harris and Bair [12], from work with three falling body viscometers with uncertainties between ±(2 and 4)%, cover temperatures between (273 and 373) K and pressures below 1 GPa.

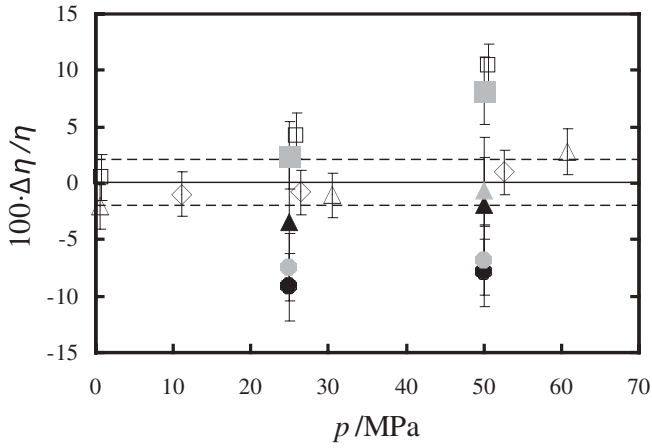

FIGURE 3
Relative deviations $\Delta\eta/\eta = \{\eta(\text{expt}) - \eta(\text{calc})\}/\eta(\text{calc})$ of the experimentally determined viscosity $\eta(\text{expt})$ from the value $\eta(\text{calc})$ obtained from equation (5) of reference [11] with coefficients of Table 4 of reference [11] as a function of pressure p at temperatures and pressures that overlap those of reference [11]. □, Harris and Bair [12] at $T = 313\,\text{K}$; △, Harris and Bair [12] at $T = 338\,\text{K}$; ◇, Harris and Bair [12] at $T = 348\,\text{K}$; ■, Harris and Bair [12] at $T = 313\,\text{K}$; ▲, Harris and Bair [12] at $T = 338\,\text{K}$; ●, Harris and Bair [12] at $T = 373\,\text{K}$; ■, Harris and Bair [12] at $T = 313\,\text{K}$; ▲, Harris and Bair [12] at $T = 338\,\text{K}$; ●, Harris and Bair [12] at $T = 373\,\text{K}$. The dashed lines are the expanded uncertainties in the measurements of reference [11] and the error bars are those cited in reference [12]. Reproduced with permission of the American Chemical Society [11].

The figures 2 and 3 illustrate the good agreement between the various sources of experimental data for the viscosity of DIDP and that agreement has prompted the development of a standard value for the viscosity of DIDP that is being prepared for publication by the International Association for Transport Properties. At least in part this work satisfies the requirement, for example, of the petroleum industry for a calibrant at temperatures up to 473 K and pressures below 200 MPa [15,16].

3 MEMS MEASUREMENT OF DENSITY AND VISCOSITY FOR NEWTONIAN FLUIDS

To develop a method for *in situ* simultaneous measurement of both density and viscosity in extreme environments another vibrating object was fabricated using the methods of MEMS. The geometry of such a vibrating device is first constrained by the requirements of being able to derive working equations for a well-defined shape of known dimensions. Secondly, when fabricated by MEMS, the practical constraints necessarily precluded the use of curved surfaces. One of the devices developed is the transducer shown in Figure 4. This device is similar to a cantilever in that it consists of a rectangular plate

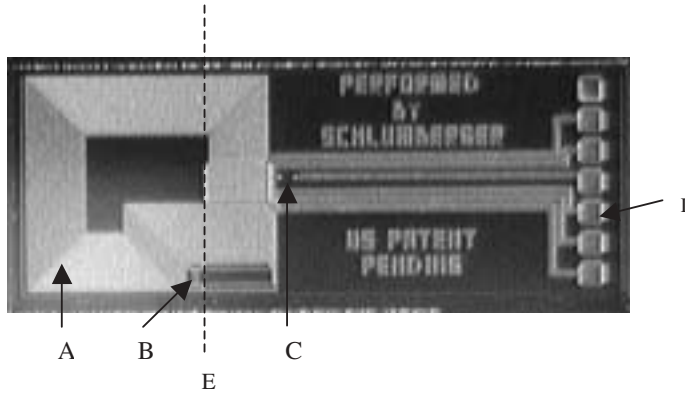

FIGURE 4
Photograph of the top surface of the MEMS showing the aluminum coil A, Wheatstone bridge B, boron doped polycrystalline silicon resistor C that acted as thermometer, and wire-bond pads D. The plate to be vibrated of thickness about 22.25 μm lies to the left of dashed line E. To the right of the dashed line E the MEMS is about 373 μm thick of which 350 μm is mono-crystalline silicon. Reproduced with the permission of the American Chemical Society [17].

connected to a support along one edge, but dissimilar in that the aspect ratio of width-to-length is greater than unity. The device described here is made of silicon and has a width of about 2 mm and a length of about 1.5 mm. The device has plate thickness of about 20 μm. Further details of the MEMS edge-supported vibrating plate densimeter/viscometer are provided by Goodwin *et al*. [17].

3.1 Working equations

The general effect of the fluid on the plate can be understood by two straightforward approximations. First, the resonant frequency decreases with increasing density because of added mass. Second, the resonance quality factor Q (defined in Reference [17]) decreases as the viscosity increases because of the shearing motion at the tip of the plate. The model, presented in detail in reference [17] (and in reference [18], where the influence of Brownian dynamics on a plate oscillating near a wall are also considered), describes oscillations of the plate in an inviscid fluid. This model therefore decouples the effects of viscosity and density so that the density is determined solely from the resonance frequency, and an independent equation is used to determine the viscosity. The plate is modeled as a beam that is supported at one end (denoted by $z = y = 0$). We assume that longitudinal strain varies linearly across the depth of the plate and the bending moment at any cross section is proportional to a local radius of curvature. We consider only the first eigenmode (1,0) so that terms containing derivatives in x can be discarded. The partial differential equation for the beam displacement when a force F is applied normal to the plate can then be estimated from the Bernoulli-Euler bending theory of thin plates. In this analysis, we have assumed the fluid is inviscid and

incompressible and the boundary conditions are pinned or simply-supported. The equation of motion for the displacement $q(z)$ of the beam is therefore given by

$$\rho_S d \frac{\partial^2 q}{\partial t^2} + D\nabla^4 q = F, \tag{1}$$

where t denotes time, $\nabla^4 = \partial^4/\partial z^4$, d the thickness of the plate, ρ_S the density of the plate material and $D = Ed^3/\{12(1-\sigma^2)\}$ is the flexural rigidity of the plate. Here E is Young's modulus and σ Possion's ratio for the plate material. At the supported end ($y = z = 0$) the oscillating plate has neither deflection nor bending so that $q = \partial q/\partial z = 0$, while at the free end of the plate ($z = a$) (furthest from the support) there is neither bending nor shear force so $\partial^2 q/\partial z^2 = \partial^3 q/\partial z^3 = 0$. When we assume there is no force acting on the plate (i.e. in vacuo) and the plate oscillates with harmonic motion, the eigenvalues υ_n satisfy $\tan(a\upsilon_n) = \tanh(a\upsilon_n)$ (so that the first eigenvalue $\upsilon_1 = 3.926\,602\,312$) provided that $48\rho_s\pi^2 f^2 a^4(1-\sigma^2) = Ed^2\upsilon_n^4$ (see below for the definition of the parameters). The MEMS plate is about 2×10^{-5} m thick, (assuming uniformity along its length) and about 1.5×10^{-3} m long. It thus satisfies the slender body criteria which can therefore be applied to the oscillations of the plate and the plate boundary.

We further assume that the force acting on the plate is equal to the pressure difference across the plate, and that the pressure, for an inviscid fluid, neglecting gravity and the velocity terms, can be obtained from Bernoulli's equation. The expression for the force is then used to find a solution that satisfies both plate and fluid equations of motion to give

$$\rho = \frac{E\upsilon_n^5 d^3}{24\{1-\sigma^2\}a^5(2\pi f)^2} - \frac{\rho_s d\upsilon_n}{2a}, \tag{2}$$

where ρ is the fluid density and f is the resonance frequency of the plate immersed in the fluid. Equation (2) requires values of E, σ, and ρ_S for the 20 μm thick silicon plate with an additional thickness of about 2.3 μm consisting of silicon nitride, silicon oxide, and aluminium. These parameters were assumed equal to those of silicon and are described in reference [17].

Using the known physical properties of Silicon for $T = 323$ K, equation (2) was used to estimate the resonance frequency of the plate when the MEMS device was exposed to argon at $T = 323$ K and at 11 pressures between (7 and 68) MPa. The ratio of the calculated to measured frequency was found to be about 4.8. These rather large differences between the estimated and measured frequencies must arise from a combination of practical departures from the assumptions used to derive Equation (1) and the assumption that the physical properties of the plate are those of silicon. Almost all of the effects will be associated with extra dissipation and extra mass in the moving components (such as the silica mounting of the plate) and so it is to be expected that the practical resonance frequency will be below that expected for the ideal sensor.

Consequently, two additional parameters C_1 and C_2, which are determined by calibration, are included in Equation (2) so that that it becomes

$$\rho = \frac{C_1 E \upsilon_n^5 d^3}{24\{1-\sigma^2\}a^5(2\pi f)^2} - \frac{C_2 \rho_s d \upsilon_n}{2a}. \tag{3}$$

The fluid viscosity η was determined from an equation given elsewhere [17]

$$\eta = \frac{C_3}{\rho f^3}[Q^{-1} - \{Q(p=0)\}^{-1}]^2, \tag{4}$$

where f is the resonance frequency of the plate immersed in the fluid, Q is the resonance quality factor in fluid, $Q(p = 0)$ in a vacuum, and ρ is the fluid density obtained from equation (4), and C_3 is a further constant determined by calibration. Equations (3) and (4) are the working equations for the measurements.

3.2 Results

The complex resonance frequency of the first eigenmode, which is a symmetrical bending mode with flexural motion, of the edge supported plate were measured while it was immersed in octane at temperatures between (323 and 423) K and $p \leq 68$ MPa. The density and viscosity were obtained from equations (3) and (4), respectively combined with calibration coefficients C_1, C_2 and C_3 obtained when the plate was immersed in methylbenzene at a temperature of 323 K reference [17]. The density obtained from equation (2) with the MEMS was used in equation (4) to determine the viscosity. For both density and viscosity the major source of uncertainty arises from the uncertainty of the calibration. And they are, based on the absolute average errors, $< \pm 0.1\%$ for density and $< \pm 1.4\%$ for viscosity.

The density of octane was determined at four temperatures of (323, 348, 373, and 423) K at $p \leq 68$ MPa, with a device that had been calibrated with methylbenzene at $T = 323$ K previously. Figure 5 shows the results as relative differences from the density, ρ, obtained from a Helmholtz function correlation of literature values reported by Span and Wagner [19] as a function of pressure p. The correlation of Span and Wagner [19] has an uncertainty of $\pm 0.2\%$ at $p \leq 30$ MPa and $\pm 0.5\%$ at $p > 30$ MPa; that is depicted in Figure 3 with broken lines. The results are within the combined uncertainty of the measurements and the values from Reference [19].

The measurements of the viscosity of octane are compared, in Figure 6, with values obtained from a correlation of values reported in the literature by Huber *et al.* [20]. At $T < 423$ K the results agree with the correlation within the combined uncertainty. At $T = 423$ K our results have an expanded uncertainty of about $\pm 6\%$ and differ by between (6 and 14)% from reference [20]. The viscosity reported by Caudwell [21], and obtained with a vibrating wire viscometer, agree with those obtained from the MEMS device within the estimated expanded uncertainty of our results.

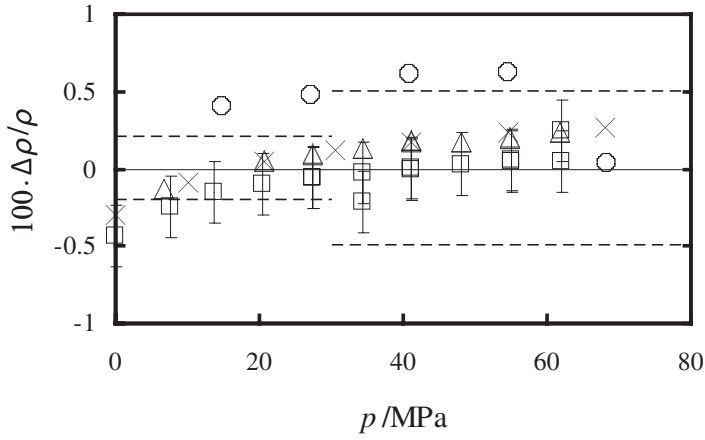

FIGURE 5
Fractional deviations $\Delta\rho/\rho = \{\rho(\text{expt}) - (\text{calc})\}/\rho(\text{calc})$ of the experimental densities $\rho(\text{expt})$ of octane reported in reference [17] from values $\rho(\text{calc.})$ obtained with the correlation of Span and Wagner [19] as a function of pressure p. ×, $T = 323.16$ K; □, $T = 348.15$ K; Δ, $T = 373.15$ K; ◯, $T = 423.15$ K; and ----, uncertainty of the correlations Refs. [19] and [20]. Reproduced with permission of the American Chemical Society [17].

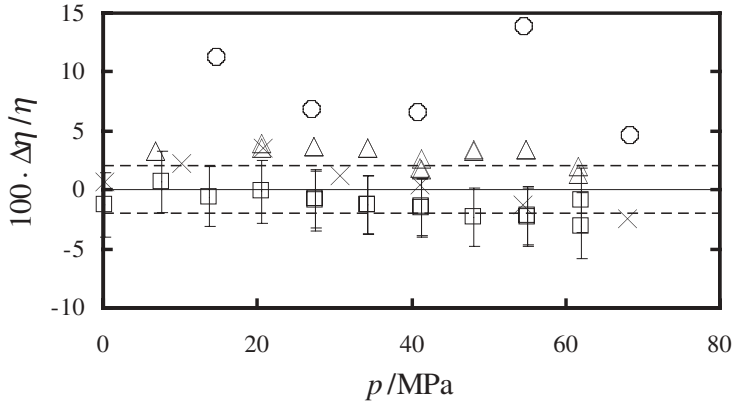

FIGURE 6
Fractional deviations $\Delta\eta/\eta = \{\eta(\text{expt}) - \eta(\text{calc})\}/\eta(\text{calc})$ of the experimental viscosities $\eta(\text{expt})$ of octane from reference [17] from values $\eta(\text{calc.})$ obtained with the correlation of Huber *et al.* [20] as a function of p. ×, $T = 323.16$ K; □, $T = 348.15$ K; Δ, $T = 373.15$ K; ◯, $T = 423.15$ K; and ----, uncertainty of the correlations Refs. [17] and [20]. Reproduced with permission of the American Chemical Society [17].

It should be emphasized here perhaps that the expanded uncertainties of the data are not that usually required for measurements of the viscosity of octane, but if they could be maintained at this level some 5 km underground in an oil reservoir within a fluid of unknown composition it would be exceedingly important.

4 MEMS FOR NON-NEWTONIAN FLUIDS

Measuring the non-Newtonian properties of downhole fluids (for example, drilling lubricants that contain sodium bentonite, added to increase the density) *in situ* poses major challenges. One device that might be capable of determining both Newtonian and non-Newtonian properties of downhole fluids is a MEMS device fabricated from anisotropic single crystal silicon with crystalline direction ⟨100⟩. The MEMS fabrication process is very similar to that for the device described in Section 3 above. The dimensions of the device are shown in Figure 7. The device contains both mechanical and electrical components. The plate oscillates parallel to its plane when an alternating current is passed through an aluminium wire coil that lies on top of the plate, which is held in an externally applied magnetic field. The force that excites the oscillation may be varied by changing either the current or the intensity of the magnetic field (full details are given in reference [22]). The plate is supported by 24 legs on each of its two sides and a Wheatstone bridge strain gauge is formed from resistors deposited atop the upper six legs on each side.

4.1 Working equations

We illustrate the possibilities of this new device by considering its analysis in a viscoelastic flow. We assume that the fluid obeys the Maxwell equations

$$\frac{\partial q_i}{\partial t} + q_j \frac{\partial q_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{\partial \sigma_{ij}}{\partial x_j} \quad \text{and} \quad \frac{\partial q_j}{\partial x_j} = 0, \tag{5}$$

where the fluid stress tensor σ_{ij} is given by

$$\frac{\partial \sigma_{ij}}{\partial t} + q_k \frac{\partial \sigma_{ij}}{\partial x_k} - \frac{\partial q_i}{\partial x_k}\sigma_{kj} - \frac{\partial q_j}{\partial x_k}\sigma_{ki} - \frac{1}{\theta}\sigma_{ij} = \frac{\nu}{\theta}\left(\frac{\partial q_i}{\partial x_j} + \frac{\partial q_j}{\partial x_i}\right), \tag{6}$$

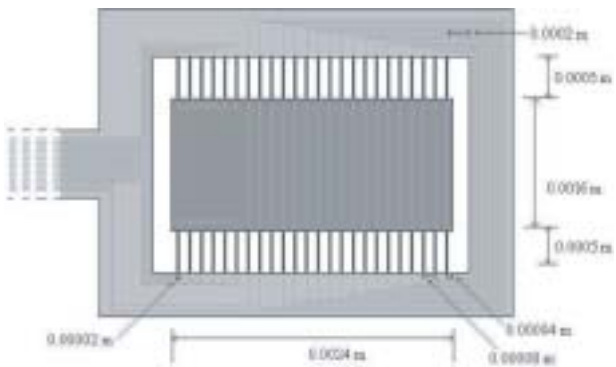

FIGURE 7
Schematic of the top surface of a transversely oscillating MEMS device (dimensions included). The light grey area is about 0.35 mm thick while the dark grey, which is the moving element, has a thickness of about 20 μm. The medium grey lines represent aluminium interconnecting wires.

and $\theta = \upsilon/G$ where G is the shear modulus of the fluid. We assume that, in the absence of an imposed pressure gradient, the fluid above and below the MEMS device executes a plane parallel shear flow with fluid velocity $\boldsymbol{q}(x, y, z, t) = (u(y, t), 0, 0)^T$. The equations of motion therefore reduce to

$$\frac{\partial^2 u}{\partial t^2} + \frac{1}{\theta}\frac{\partial u}{\partial t} = \frac{\upsilon}{\theta}\frac{\partial^2 u}{\partial y^2}, \tag{7}$$

with boundary conditions $u(0, t) = U_p(t) = U_0 \,\mathrm{Re}(e^{-i\omega t})$, $u \to 0$ as $y \to \infty$. The governing equation may now be solved for $u(y, t)$ to yield (having taken real parts)

$$\begin{aligned} u = U_0 \exp\{-\sqrt{\omega/\upsilon}(1 + \omega^2\theta^2)^{1/4} y \sin(\gamma/2)\} \\ \times \cos\{\sqrt{\omega/\upsilon}(1 + \omega^2\theta^2)^{1/4} y \cos(\gamma/2) - \omega t\}, \end{aligned} \tag{8}$$

where $\gamma = \arctan(1/\omega\theta)$. For simplicity, let us first examine how the theory works for a linear viscous fluid. In this case, equation (8) reduces to

$$u = U_0 \exp(-y/\delta)\cos(y/\delta - \omega t), \tag{9}$$

where $\delta = (2\upsilon/\omega)^{1/2}$ is the viscous penetration depth. The power P required to move the plate is given by

$$P|_{y=0} = \frac{\omega}{2\pi}\left|\int_0^{2\pi/\omega} Su|_{y=0}\mathrm{d}t\right|, \tag{10}$$

where S is the frictional force acting in the x-direction. For a linear viscous fluid S is given by

$$S = 2aB[\sigma_{xy}]|_{y=0} = 2aB\left[\eta\frac{\partial u}{\partial y}\right]_{y=0}, \tag{11}$$

where a and B are respectively the length and width of the oscillating plate so that

$$\eta = \frac{2}{\omega\rho}\left[\frac{|P|_{y=0}|}{aBU_0^2}\right]^2. \tag{12}$$

In principle, this allows the fluid viscosity η to be determined provided the device power P and the speed U_0 of the plate are both known. For a viscoelastic fluid, matters are complicated slightly by the fact that to determine the shear stress of the fluid the equation

$$\frac{\partial \sigma_{xy}}{\partial t} + \frac{\sigma_{xy}}{\theta} = \frac{\upsilon}{\theta}u_y, \tag{13}$$

must now be solved. Notwithstanding this, it is possible to derive an expression for the power on the plate. Though the form of the solution does not allow

a closed-form solution for the viscosity, the resulting equation can easily be solved numerically and once again the viscosity may be determined if the power and the plate displacement are both known. Preliminary attempts to operate the device have resulted in mechanical failures that have prevented acquisition of data to validate the model. Subsequent analyses have shown these failures arise from both fractures and complete breaks in one or more of the 48 legs, which connect the plate to the support. The data acquired thus far and the theory now available will be used to revise the design and fabricate a device with fewer and more compliant legs.

5 CONCLUSIONS

For the three cases described, robustness of the device was given a higher priority in the design than that assigned to accuracy. Each instrument has working equations founded on the principles of physics. The vibrating wire provided measurements of the viscosity of DIDP that met the accuracy requirement of $\pm 1\%$ while the MEMS densimeter/viscometer achieved the goal of determining density to $\pm 1\%$ and viscosity to $\pm 10\%$; the MEMS fabrication techniques provided a densimeter very sensitive to added mass, and thus density, because the plate has a large surface-to-volume ratio and a mass of only 0.12 mg.

NOMENCLATURE

T	Temperature
p	Pressure
Q	Resonance quality factor
z	direction
y	direction
F	Force
q_z	beam displacement
t	time
d	plate thickness
ρ_s	density of the plate material
D	flexural rigidity
E	Young's modulus
σ	Possion's ratio
υ_n	eigenvalue
ρ	fluid density
f	resonance frequency
η	dynamic viscosity
σ_{ij}	fluid stress tensor
G	shear modulus
υ	kinematic viscosity

$\boldsymbol{q}$	fluid velocity
u	velocity component
U_0	vibrating plate speed
$\theta = \upsilon/G$	
ω	angular frequency ($= 2\pi f$)
δ	viscous penetration depth
P	power
S	frictional force in the x direction
a	length of the oscillating plate
B	width of the oscillating plate
η(expt)	experimental viscosity
η(calc)	calculated viscosity
ρ(expt)	experimental density
ρ(calc)	calculated density

REFERENCES

[1] Wakeham W. A., Assael M. A., Atkinson J. S., Bilek J., Fareleira J. M. N. A., Fitt A. D., Goodwin A. R. H. and Oliveira C. M. B. P. *Int. J. Thermophys.*, **28** (2007), 372.

[2] Künzel W., Van Wijk H. F. and Marsh K. N. *Viscosity*. In *Recommended Reference Materials for the Realization of Physicochemical Properties*, Marsh K. N., Ed. (IUPAC), Oxford, UK: Blackwells, 1987, 45.

[3] Viscosity of water. ISO/TR 3666: 1988.

[4] Caetano F. J. P., Fareleira J. M. N. A., Oliveira C. M. B. P. and Wakeham W. A. *Int. J. Thermophys.*, **25** (2004), 1311.

[5] Retsina T., Richardson S. M. and Wakeham W. A. *Appl. Sci. Res.*, **43** (1986), 127.

[6] Retsina T., Richardson S. M. and Wakeham W. A. *Appl. Sci. Res.*, **43** (1987), 325.

[7] Caetano F. J. P., Fareleira J. M. N. A., Oliveira C. M. B. P. and Wakeham W. A. *J. Chem. Eng. Data.*, **50** (2005), 201.

[8] Kandil M. E., Marsh K. N. and Goodwin A. R. H. *J. Chem. Eng. Data.*, **50** (2005), 647.

[9] Caetano F. J. P., Fareleira J. M. N. A., Oliveira C. M. B. P. and Wakeham W. A. *J. Chem. Eng. Data.*, **50** (2005), 1875.

[10] Caetano F. J. P., Fareleira J. M. N. A., Fernandes A. C., Oliveira C. M. B. P., Serro A. P., Simões de Almeida I. M. and Wakeham W. A. *Fluid Phase Equilib.*, **245** (2006), 1.

[11] Al Motari M. M., Kandil M. E., Marsh K. N. and Goodwin A. R. H. *J. Chem. Eng. Data.*, **52** (2007), 1233.

[12] Harris K. R. and Bair S. *J. Chem. Eng. Data.*, **52** (2007), 272.

[13] Peleties F. and Trusler J. P. M. (2006), Private communication.

[14] Vogel H. *Physik Z.*, **22** (1921), 645.

[15] Lundstrum R., Goodwin A. R. H., Hsu K., Frels M., Caudwell D. R., Trusler J. P. M. and Marsh K. N. *J. Chem. Eng. Data.*, **50** (2005), 1377.

[16] Sopkow T., Goodwin A. R. H. and Hsu K. *Chem. Eng. Data.*, **5** (2005), 1732.

[17] Goodwin A. R. H., Donzier E. P., Vancauwenberghe O., Manrique de Lara M., Marty F., Mercier B., Fitt A. D., Ronaldson K. A. and Wakeham W. A. *J. Chem. Eng. Data.*, **51** (2006), 190.

[18] Clarke R. J, Jensen O. E., Billingham J., Pearson A. P. and Williams P. M. *Phy. Rev. Lett.*, **96** (2006), 050801.

[19] Span R. and Wagner W. *Int. J. Thermophys.*, **24** (2003), 41.

[20] Huber M. L., Laesecke A. and Xiang, H. W. *Fluid Phase Equilib.*, **224** (2004), 263.

[21] Caudwell D. R. *PhD thesis*. University of London, London; 2004.

[22] Ronaldson K. A., Fitt A. D., Goodwin A. R. H. and Wakeham W. A. *Int. J. Thermophys.* **27** (2006), 1677.